Probabilistic Modeling of Power lines Magnetic Fields

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Abstract: The transmission line conductor sag has a direct effect on the intensity of magnetic fields in the neighborhood of power lines. Several factors are known to contribute to the temperature of the power line conductors which, in turn, influences the resulting line sag, namely, line loading, ambient temperature, solar irradiation, and wind speed. The probabilistic nature of those factors will reflect on the line sag and consequently on the prevailing magnetic fields. This paper considers those factors and examines possible random variations in their values and the subsequent effect on magnetic fields.

Keywords: magnetic fields, line conductor sag, probabilistic modeling

I. INTRODUCTION

To obtain accurate information on the possible biological effects of electric power systems and installations, many computational efforts were employed to evaluate magnetic fields in the vicinity of different power line configurations [1-3]. A complex magnetic field environment exists near a power line, where a number of factors are known to affect the magnetic field intensities in a direct or indirect manner. For a certain power line configuration, the magnetic field intensity at a given position in the line vicinity, and at the standard height of one meter above the ground, two main factors affect magnetic fields: line current (or, loading) and conductor sag which, in turn, determines the conductor height above the ground. The conductor sag is determined by those factors which influence conductor heating (or, cooling), namely, line loading, ambient temperature, wind speed, and solar irradiation (direct sunshine). The evaluation of sag variations due to conductor heating is the hot topic of today's power industry caused by the tendency towards transmission lines uprating [4].

The present paper, in an effort to model magnetic field variations accurately, links the conductor sag to the above factors. The random nature of those factors is then recognized and investigated. The overall probabilistic effect on magnetic fields in the power line neighborhood is then modeled.

II. FACTORS INFLUENCING CONDUCTOR SAG

The conductor sag under known conditions of loading, ambient temperature, wind speed, and solar irradiation can be calculated by equating the heat developed in the conductor with the dissipated power from the conductor. The heat is developed in the conductor by the I^2R loss and solar irradiation, while heat is dissipated by convection in the surrounding air and radiation to surrounding objects [5]. This can be expressed as follows:

$$W_i + W_s = (W_c + W_r)\,A \qquad (1)$$

where, A is the conductor surface area in m^2 per meter of conductor length ;

$W_i = I^2 R_m$ W/m ,

R_m being the resistance of the conductor per meter length at any temperature.
W_s is the power developed in the conductor by solar irradiation, where,

$$W_s = S_a\, i\, d \qquad \text{W/m} \qquad (2)$$

d is the conductor diameter in meter, i is the prevailing solar irradiation intensity in W/m^2, and S_a is the solar absorption coefficient which equals 1.0 for a black body, or well-weathered conductor, and equals about 0.6 for new conductors. The power dissipated by convection, W_c, can be determined as follows:

$$W_c = \frac{5.73\sqrt{ps}}{T_a^{0.123}\sqrt{d}}\Delta T \qquad \text{W/m}^2 \qquad (3)$$

where, p is the pressure (in atmospheres), s is the air speed (in m/s), T_a is the average absolute temperature in °K, d is the conductor diameter (m), and ΔT is the temperature rise of conductor above ambient.
The power dissipated by radiation, can be determined as follows:

$$W_r = 57000\,E\left[\left(\frac{T}{1000}\right)^4 - \left(\frac{T_o}{1000}\right)^4\right] \qquad \text{W/m}^2 \qquad (4)$$

where, E is the relative emissivity of conductor surface, which equals 1.0 for black body, and about 0.5 for average

oxidized copper and aluminum, T is the absolute temperature of conductor and T_o is the absolute ambient temperature.

The subsequent change in the conductor length can be calculated as follows:

$$\Delta L = \Delta T \cdot \alpha \cdot L \qquad (5)$$

where L is the conductor length and α is the coefficient of linear expansion of the conductor.

A case study involving a double-cicuit 220 kV line which uses AAAC conductors with bundle sizes of 2x405 mm^2 and 30 cm bundle spacing is used in this work. The full load current per phase of the line is 1200A and the maximum sag is 11.5 m at 80°C conductor temperature and 45°C ambient temperature, as specified by Egyptian standards. In Fig. 1 the influence of conductor sag on magnetic field is shown at mid-span where it is evaluated at the power line's center-line and at the right-of-way. Also shown is the absolute maximum magnetic field, which approximately occurs beneath the outer phase conductor.

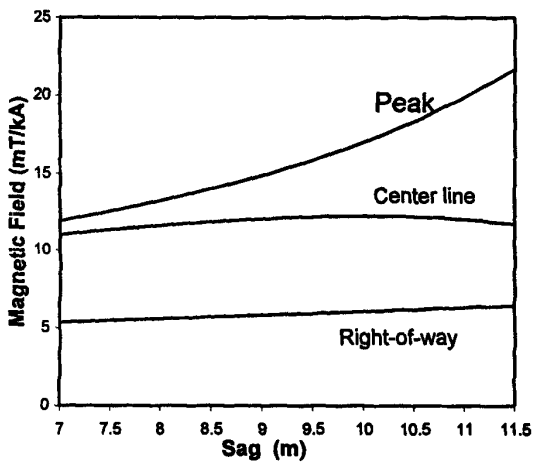

Fig. 1 Magnetic field versus conductor sag

III. PROBABILISTIC FEATURES

The variations in each and every one of the factors influencing the sag ,and consequently the magnetic field, are basically deterministic in their own right since they are direct products of natural or physical mechanisms. However, when the factor is monitored over a long enough period those variations may be globally looked upon probabilistically. This applies to line loading, ambient temperature, wind speed, and solar irradiation energy. Data were acquired for the study case by the Egyptian Electricity Authority and the New and Renewable Energy Authority.

A. Statistical Distribution of Load

Load fluctuations are normally expressed in the form of a daily (weekly, or annual...) load curve which gives the pattern of load change with time over the period of concern. The load-duration curve, meanwhile, expresses the time over which a given load level, or higher, prevails. If the load is treated as a random variable the distribution of the load's frequency-of-occurrence may be sought. The load-duration curve amounts to an expression of the cumulative probability distribution of the load. Therefore, the probability density (or, frequency) distribution is -in fact- the derivative of the load-duration relation w.r.t. load.

In Fig. 2 the load's frequency-of-occurrence distribution is directly constructed from an acquired daily load curve for the study case. The full range of load is divided into a large number of discrete loads. For each infinitesimal load range the per-unit time over which that load prevailed, i.e., the frequency of occurrence, is estimated and plotted. Depending on application, load data may be gathered over longer periods, e.g., a week or several weeks.

B. Statistical Distribution of Ambient Atmospheric Quantities

Similar to the statistical model of load the variations of ambient temperature, wind speed, and solar irradiation may be acquired for one day, processed, and their frequency distributions produced. For the present case study, the frequency distributions of the three quantities are shown in Figs. 3, 4, and 5, respectively. In Fig. 5 the very high frequency corresponding to negative irradiation accounts for night time and is, therefore, ignored in subsequent modeling.

IV. MODELING

It should be pointed out that statistical correlation may exist between any two variables, particularly when the statistical model is based on short-term data. For example, load variations may assume a pattern which is not totally independent of the pattern of ambient temperature variation since they are both –somehow- geared to the time of the day. Indeed, for the case study and based on the readings of one day, the correlation coefficient between loading and ambient temperature was found to be +0.605, which is significant in view of the size of the data sample. The correlation coefficient between loading and solar irradiation was found to be only + 0.317, while the correlation coefficient between ambient temperature and solar irradiation was +0.608.

Based on the above, the probabilistic problem of magnetic fields may be addressed using either one of two models, namely, the correlative model and the non-correlative model. The former recognizes the correlation among the four variables, specifically those between line current and ambient temperature, and between line current and solar

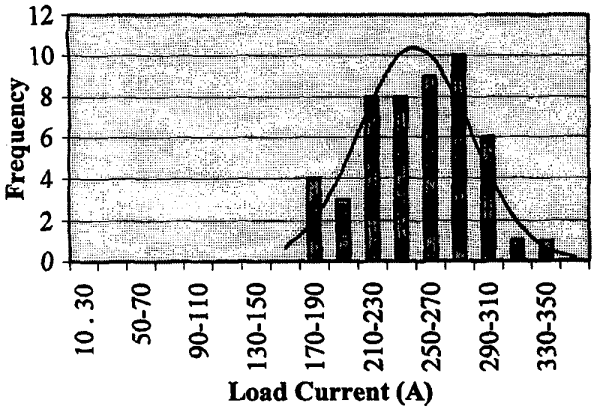

Fig. 2 Relative frequency distribution of line current

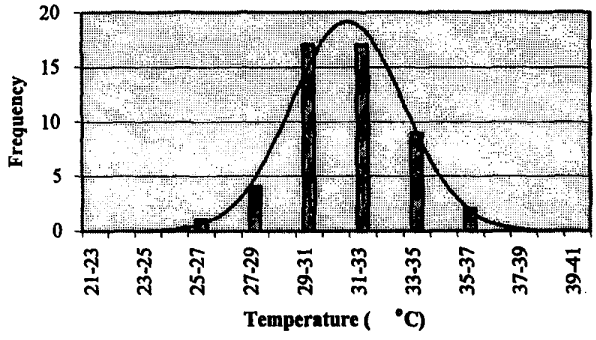

Fig. 3 Relative frequency distribution of ambient temperature

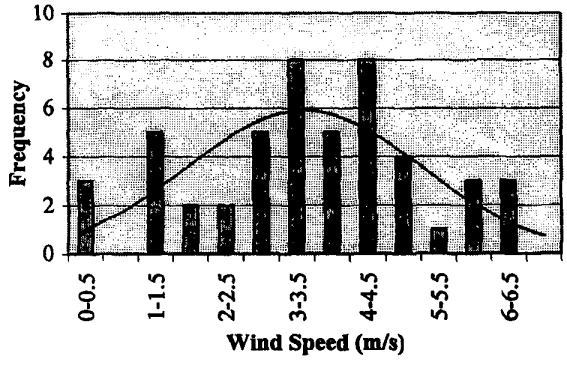

Fig. . 4 Relative frequency distribution of wind speed

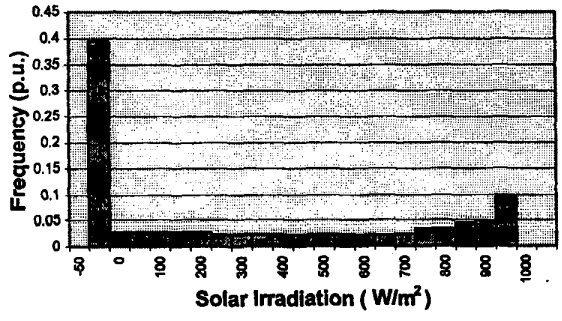

Fig. 5 Relative frequency distribution of solar irradiation

irradiation. This approach is more accurate, yet it involves more computational effort to carryout. In either model, a Monte Carlo approach is used in which random values of the variables are produced by random number generators (RNG) in accordance with their assumed probability distributions and the correlation –if any- among them.

A. Non-Correlative modeling

This model assumes that there is enough justification to ignore any inherent correlation among the four variables, i.e., their randomness is assumed to be independent. In such a case, the statistical distribution of magnetic field is produced by repeatedly generating random values for the four variables which will be used to estimate the conductor sag and subsequently the magnetic field.

Fig.6 diagrammatically illustrates the procedure of generating random magnetic field values in a non-correlative model. In the figure, c symbolizes the line current, t the ambient temperature, i the solar irradiation , and s the wind speed.

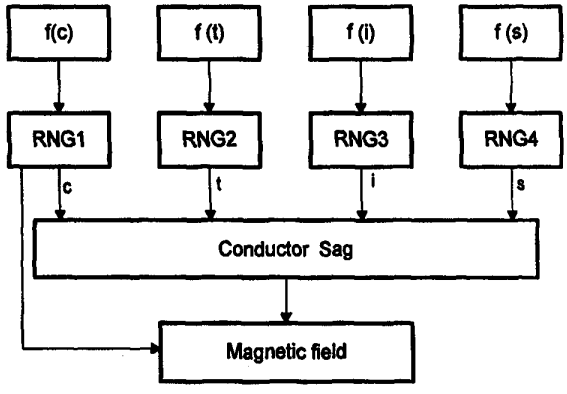

Fig. 6 Non correlative estimation of magnetic field statistics

Application to long-term planning

The non-correlative model is generally suitable for cases of time spans long enough to subdue interdependence among the four variables. Over such long time spans –typically yearly models- the statistical distributions of the variables reach more-or-less stable forms. This is useful for long-term planning.

According to the probability theory's "central limit theorem" accumulating large numbers of random variables leads to a result, which takes on the shape of a normal (gaussian) distribution given by the double-statistic expression

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (6)$$

in which μ is the mean and σ^2 is the variance (σ is the standard deviation). For the case study, and based on data accumulated over a long period of time Table 1 lists the

statistics of the distributions of the four variables for the case study. Also given in the table are the model-derived statistics of the conductor sag and the subsequent statistics of the maximum RMS magnetic field.

Table 1
Statistics of long-term distributions

Variable	Mean	Standard Deviation
Current (A)	249.6	35.0
Temprature (C)	31.2	2.3
Wind Speed (m/s)	3.55	1.68
S. Irradiation (W/m^2)	330.1	391.3
Conductor Sag (m)	8.88	0.34
Magnetic Field (μT)	3.69	0.58

Application to short-term planning
The non-correlative model may ,nevertheless, be used for shorter periods of time. Although over such periods correlation among the variables does exit the non-correlative model offers a good approximation of the output. To make up for the inaccuracy created by ignoring correlation it is recommended –in such cases- to apply (6) using actual – rather than fitted- frequency distributions f(x).

B. Correlative Modeling

A probabilistic model of magnetic field, that accounts for the correlation among its effective components, may be primarily used for short term modeling. In such cases the variation patterns of the prevailing operational and environmental variables (c, t, i, and s) do not change and, thus, should not be ignored. The following procedure, which accounts for correlation is used to generate random sag-based magnetic field intensities:

1- One random variable is freely generated at first; in this work the line current c is chosen for this leading role.
2- Corresponding to that certain value of c, the "conditional" statistical distribution of every other variable is sought.
3- The conditional distribution of every other variable is then used –by means of another random number generator- to produce a random value for that variable.

The conditional distribution of a variable y (y is a generalization of t, i, or s) for a given value of current $c=c_1$ is given by

$$f(y|c=c_1)=\frac{f(c,y)}{f(c)} \qquad (7)$$

where, f (c,y) is the "joint distribution" of the two variables, and f (c) is the "marginal" (or, overall) distribution of c. The correlative probabilistic modeling of magnetic field is diagrammatically depicted in Fig. 7.

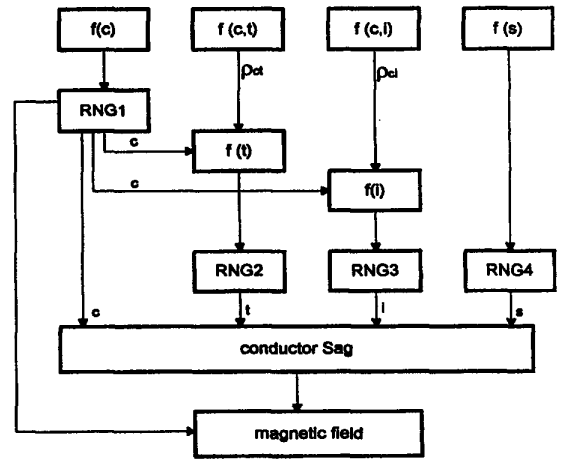

Fig. 7 Correlative estimation of magnetic field statistics

Direct application of correlative modeling
Equation (7) above may be directly, i.e. numerically applied using data of distributions of all four variables expressed in one frequency distribution array. It is more reliable when manipulating (7) numerically to deal with cumulative -rather than frequency- distributions, i.e. F(x). The target is to deduce the cumulative distribution function $F(y|c=c_1)$ from which a random value of y may be generated . This will make use of the cumulative distributions of the variable y available in the data matrix for all values of c, i.e., $F(y|c=c_i)$, $F(y|c=c_{ii})$, $F(y|c=c_{iii})$,... , where c_i, c_{ii}, c_{iii},... are the acquired data values of c. The targeted function $F(y|c=c_1)$ can then be deduced by interpolating its values for successive values of y.

Application of correlative modeling based on normalization
The statistical features of most random variables can be reliably expressed by their mean and variance only. This simple expression facilitates further treatment of the variables. In such cases the statistics of the conditional distribution $f(y|c=c_1)$ may be given by the following

$$\mu(y|c=c_1)=\mu_y+\rho_{cy}.(\frac{\sigma_y}{\sigma_c}).(c_1-\mu_c) \qquad (8)$$

and,

$$\sigma^2(y|c=c_1)=(1-\rho_{cy}^2).\sigma_y^2 \qquad (9)$$

where, $\mu(y|c=c_1)$ is the mean of the conditional distribution $f(y|c=c_1)$, μ_c and μ_y are the means of the variables' marginal distributions, $\sigma^2(y|c=c_1)$ is the variance of the conditional

distribution f $(y|c=c_1)$, σ^2_c and σ^2_y are the variances of the variables marginal distributions, ρ_{cy} is the correlation coefficient between c and y. All σ's are the corresponding standard deviation. In Figs. 8 and 9 the statistical fluctuations in the conductor sag and the resultant magnetic field are shown, respectively.

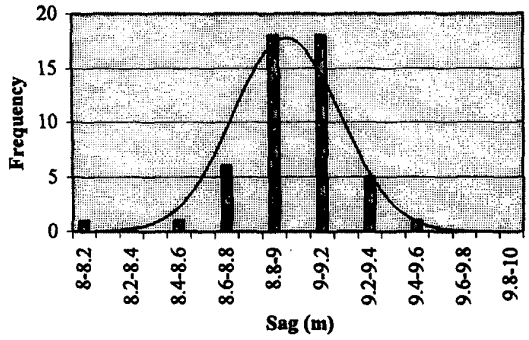

Fig. 8 Deduced relative frequency distribution of conductor sag

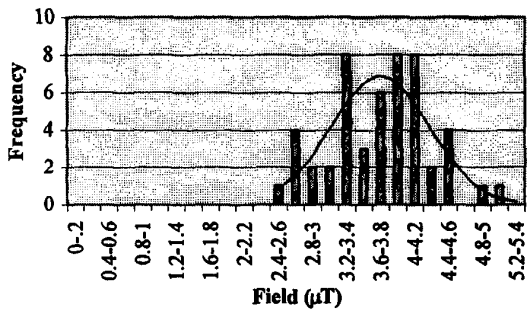

Fig. 9 Deduced relative frequency distribution of ultimate magnetic field

V. CONCLUSIONS

The present work acknowledges the probabilistic aspects of magnetic fields present in the neighborhood of high voltage power lines. The direct effect on magnetic field comes from random fluctuations in the power line current to which magnetic field is proportional. The further effects of random changes in conductor sag ,which complicates the statistical profile of magnetic field, were recognized in this work and their components were identified. Those components include –in addition to the line current- the seemingly random nature of the prevailing weather conditions, namely, atmospheric temperature, solar irradiation (or, sunshine), and wind.

The statistical features of each contributing component were produced from real data and were expressed in the form of frequency -or, probability density- distributions. Some significant statistical correlation was detected among those components, e.g., between the ambient temperature and line loading. Two models were presented, one model accounted for correlation among the variables, while the second ignored correlation in long-term applications. The collective statistical distribution of conductor sag could, subsequently, be obtained from which the ultimate statistical distribution of magnetic field could be readily deduced.

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VII. BIOGRAPHIES

H.I. Anis (SM' 79) received the B.Sc. degree from Cairo University in 1966, the M.Eng. and Ph.D. degrees from McGill University, Canada, in 1970 and 1973. He has been with Cairo University since 1974 where is now professor. He worked at, or collaborated with, numerous institutions, including IREQ institute, U. of Waterloo, and U. of British Columbia in Canada; the MIT, U. of Cincinnati, and U. of Louisiana in the USA. From 1991 through 1999 he directed a World Bank project on Engineering Education in Egypt.

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